

Soundness and Completeness

	Rough Idea	More Precise Wording	In Terms of Syntax/Semantics	In Symbols	How's the Result Demonstrated?
Soundness	Everything you can prove with proofs is true	All of the things you can prove in Fitch are things that your truth-tables would show to be tautological consequences	If P_1, \dots, P_n syntactically entails S , then P_1, \dots, P_n semantically entails S	$(P_1, \dots, P_n \vdash S) \Rightarrow (P_1, \dots, P_n \models S)$	Inductively, by looking at each truth-functional connective and seeing that its intro and elim rules preserve truth (8.3)
Completeness	Everything true can be proven by proofs	All of the things that your truth-tables show to be tautological consequences are things that you can prove in Fitch.	If P_1, \dots, P_n semantically entails S , then P_1, \dots, P_n syntactically entails S	$(P_1, \dots, P_n \models S) \Rightarrow (P_1, \dots, P_n \vdash S)$	Making something called the Henkin-construction (19); this I harder. Sometimes people use tableau's or trees.

Soundness says that all of the things you can prove in Fitch are things that your truth-tables would show to be tautological consequences. This means that all the things your truth tables would show *not* to be tautological consequences are things your Fitch proof will *not* be able to prove. Put differently: there are not things you can prove in Fitch that your truth-tables don't show to be tautological consequences.

Completeness says that all of the things that your truth-tables show to be tautological consequences are things that you can prove in Fitch. This means that all the things you *cannot* prove in Fitch are things that your truth-tables would show *not* to be tautological consequences. Put differently: there are not things that truth tables would show that our proof methods can't show.

Together soundness and completeness marry the truth tables and the proof rules. This thought is expressed more generally by saying that soundness and completeness show that the results we get by looking at syntactic things (like proof rules) are completely in sync with the results we get by looking at semantics things (like truth tables).